Transformations of Logarithmic Functions

- The graph of the logarithmic function $y = a\log_c(b(x-h))+k$ can be obtained by transforming the graph of $y=\log_c x$. These transformations should be performed in the same manner as those applied to any other function.

Example 1: Translations of a Logarithmic Function

Sketch the graph of $y=\log_4(x+4)-5$ and state the mapping rule, domain and range, x- and y- intercepts, and equation of the asymptote.

Solution:

Begin with the graph of $y=\log_4 x$. Think of $y=\log_4 x$ as $4^y = x$. Choose "nice" values of y first and then determine the x-values. Next, identify the transformations on this function to create $y=\log_4(x+4)-5$.

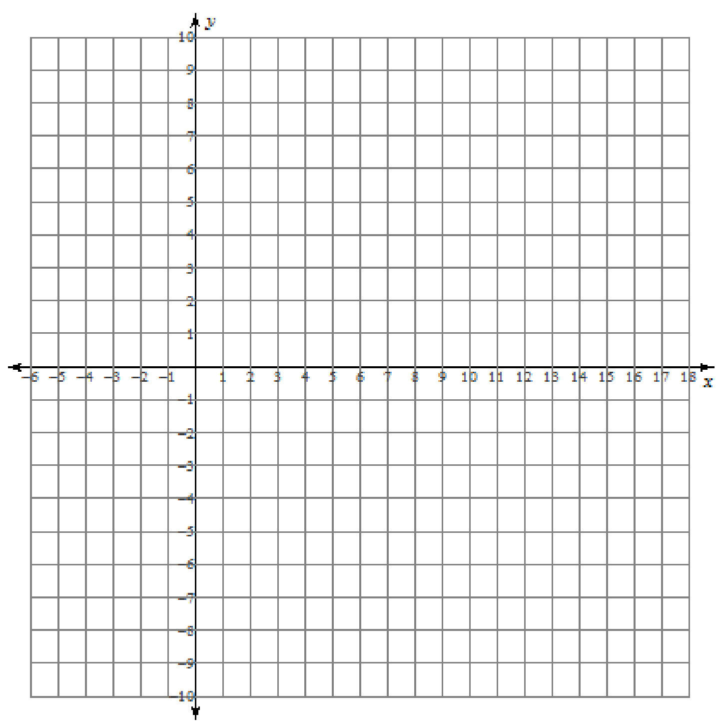
- The base graph must be translated ____.

  Mapping rule: $(x,y)\rightarrow$__________________________.

- Complete each table of values and sketch the graphs of both functions.

$y=\log_4 x$	
x	y

$y=\log_4(x+4)-5$	
x	y



For the function $y=\log_4(x+4)-5$:

Domain: ____________________________

Range: ____________________________

x-intercept: _______________

y-intercept: _______________

Equation of the vertical asymptote: ______________

Example 2: Reflections and Stretches of Logarithmic Functions

Sketch the graph of $y = -\log_2 4x$ and state the mapping rule, domain and range, x- and y- intercepts, and equation of the asymptote.

Solution:

Begin with the graph of $y = \log_2 x$. Think of $y = \log_2 x$ as $2^y = x$. Choose "nice" values of y first and then determine the x-values. Next, identify the transformations on this function to create $y = -\log_2 4x$.

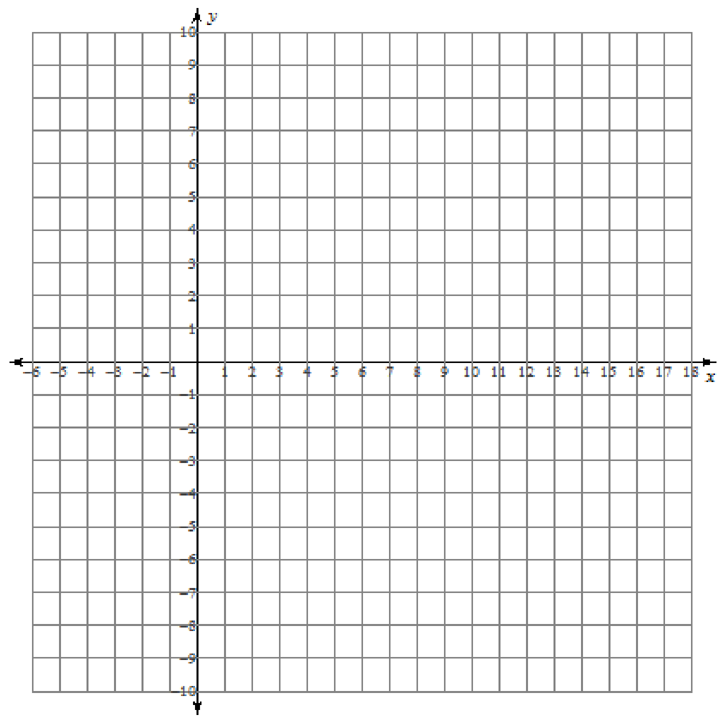
- The base graph must be ____.

  Mapping rule: $(x, y) \rightarrow$ ________________________.

- Complete each table of values and sketch the graphs of both functions.

$y = \log_2 x$	
x	y

$y = -\log_2 4x$	
x	y



For the function $y = -\log_2 4x$:

Domain: ____________________________

Range: ____________________________

x-intercept: _______________

y-intercept: _______________

Equation of the vertical asymptote: ______________

Example 3: Combine Transformations

Sketch the graph of $y = -2\log_3(x-3)+5$ and state the mapping rule, domain and range, x- and y- intercepts, and equation of the asymptote.

Solution:

Begin with the graph of $y = \log_3 x$. Think of $y = \log_3 x$ as $3^y = x$. Choose "nice" values of y first and then determine the x-values. Next, identify the transformations on this function to create $y = -2\log_3(x-3)+5$.

- The base graph must be ____

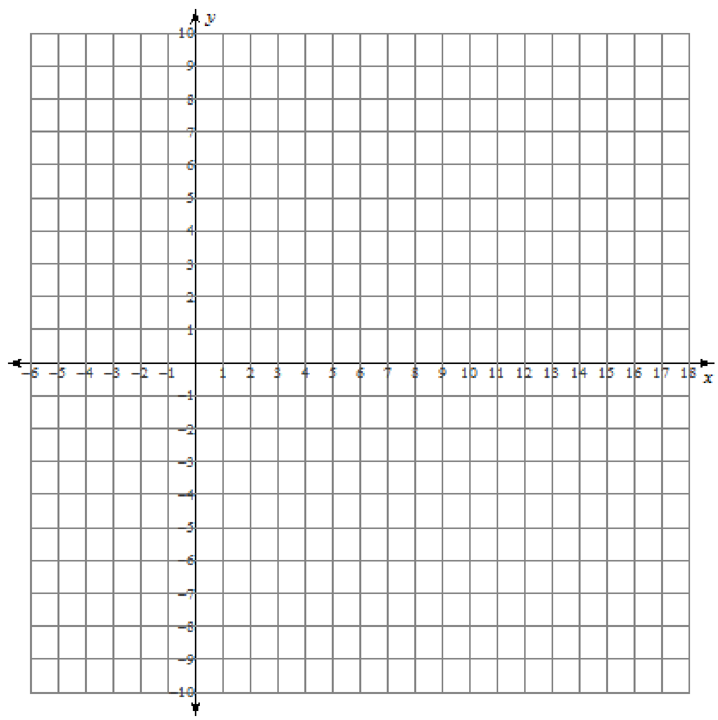
  ____.

  Mapping rule: $(x, y) \rightarrow$ ________________________.

- Complete each table of values and sketch the graphs of both functions.

$y = \log_3 x$	
x	y

$y = -2\log_3(x-3)+5$	
x	y



For the function $y = -2\log_3(x-3)+5$:

Domain: ____________________________

Range: ____________________________

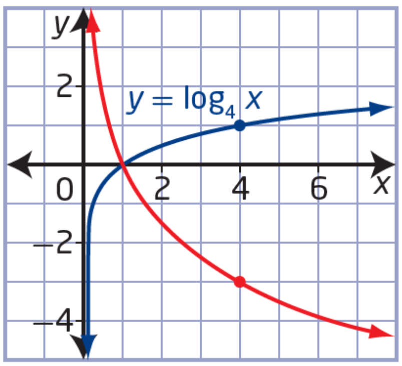
x-intercept: _______________

y-intercept: _______________

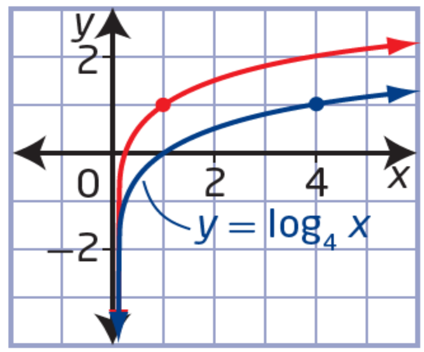
Equation of the vertical asymptote: ______________

Example 4: Determine the Equation of a Logarithmic Function Given Its Graph

a. The transformed graph illustrated in the diagram below can be generated by stretching and reflecting the graph of $y = \log_4 x$. Determine the equation of the transformed graph.



b. The transformed graph illustrated in the diagram below can be generated by stretching the graph of $y = \log_4 x$. Determine the equation of the transformed graph.



Solution:

a. ______________________________

b. ______________________________

Example 5: Use Transformations of an Exponential Function to Model a Situation

There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number, F, of flower species that a butterfly feeds on and the number, B, of butterflies observed can be modeled by the function $F = -2.641 + 8.958 \log B$.

Predict the number of butterfly observations in a region with 25 flower species.

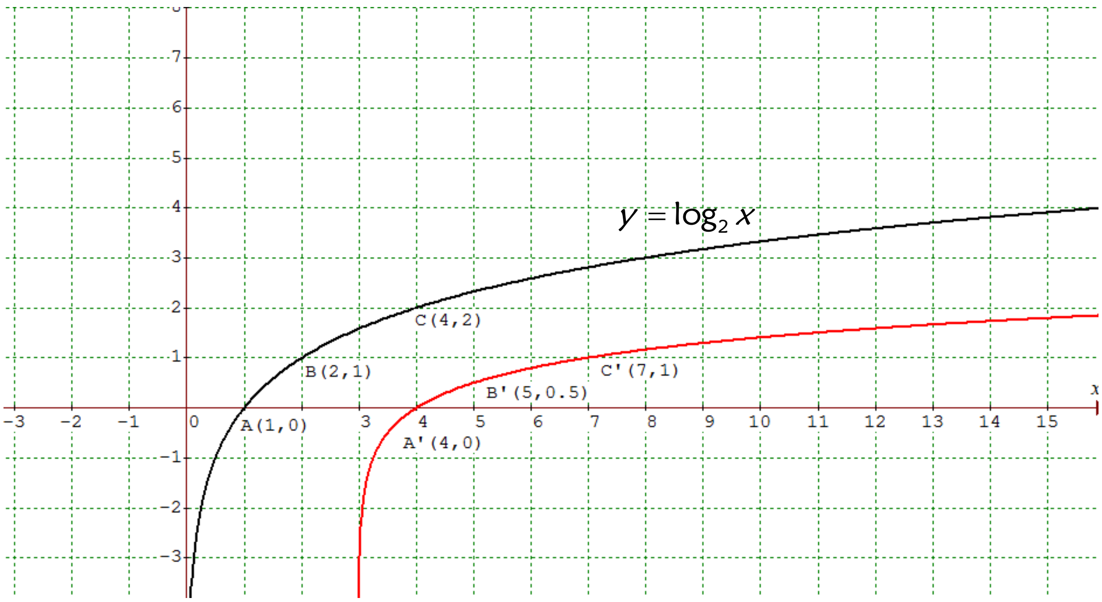
Example 6: Sketch Graphs of Transformed Logarithmic Functions

Without using a mapping rule or a table of values, sketch each of the logarithmic functions given below. Include the correct location of the vertical asymptote. For the point (1, 0) on the base function, determine the coordinates of the corresponding image point on the transformed function (Use a mapping rule for this point only).

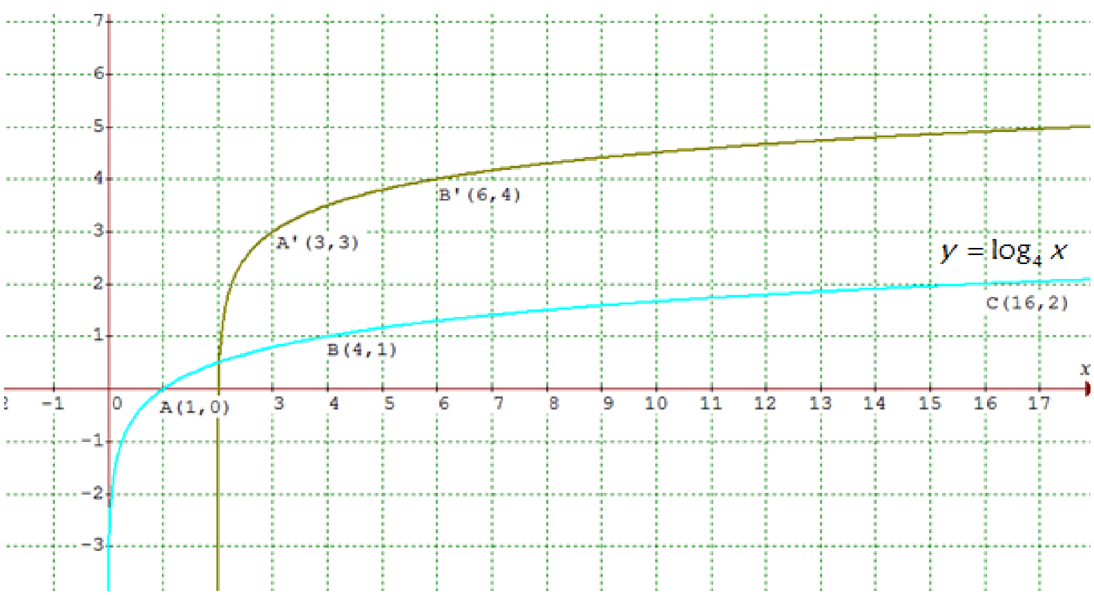
$y=\log_2\left(3(x-5)\right)+1$	$y=-\log_2\left(3(x-5)\right)+1$	$y=\log_2\left(-3(x-5)\right)+1$
$y=-\log_2\left(-3(x-5)\right)+1$	$y=2\log_{\frac{1}{3}}\left(x+4\right)-2$	$y=-2\log_{\frac{1}{3}}\left(x+4\right)-2$
$y=2\log_{\frac{1}{3}}\left(-(x+4)\right)-2$	$y=-2\log_{\frac{1}{3}}\left(-(x+4)\right)-2$	$y=-\frac{1}{2}\log_4\left(-3(x+1)\right)+3$

EXTRA PRACTICE:

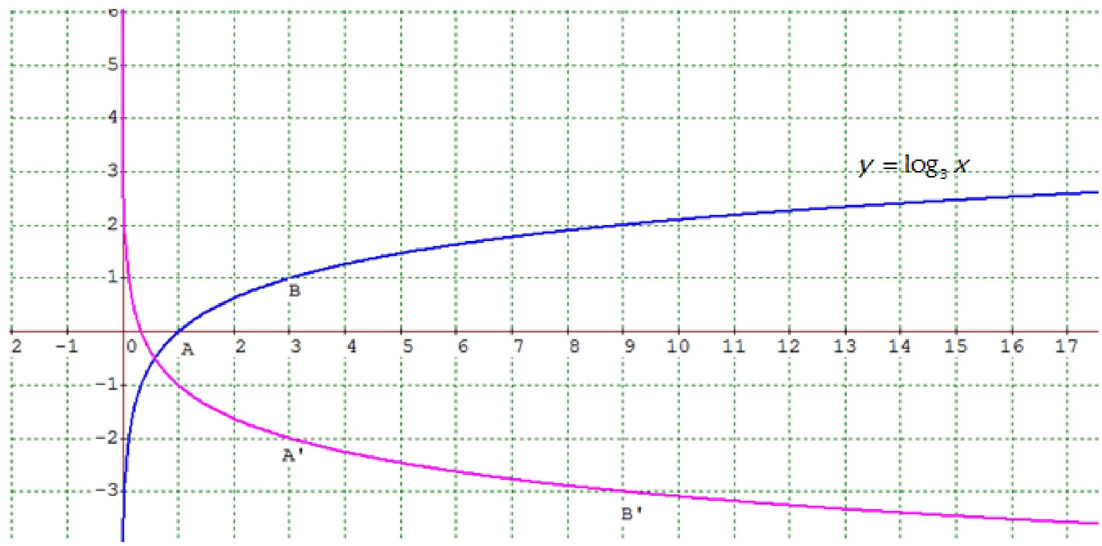
The transformed graph shown on the grid can be generated by stretching and translating the graph of $y = \log_2 x$. Determine the equation of the transformed graph.



The transformed graph shown on the grid can be generated by translating the graph of $y = \log_4 x$. Determine the equation of the transformed graph.



The transformed graph shown on the grid can be generated by reflecting, stretching and translating the graph of $y = \log_3 x$. Determine the equation of the transformed graph.



The transformed graph shown on the grid can be generated by reflecting, stretching and translating the graph of $y = \log_5 x$. Determine the equation of the transformed graph.