Proving Conjectures: Deductive Reasoning

**Proof** – A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

**Deductive Reasoning** – Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

**Example 1: Connecting Conjectures with Reasoning**

Use *inductive* reasoning to make a conjecture about the connection between the *sum* of 5 consecutive integers and the *median* of these numbers.

Use *deductive* reasoning to *prove* your conjecture.

**Solution:**

**Inductive reasoning:**

1. _____ + _____ + _____ + _____ + _____ = _____ median = _____
2. _____ + _____ + _____ + _____ + _____ = _____ median = _____
3. _____ + _____ + _____ + _____ + _____ = _____ median = _____
4. _____ + _____ + _____ + _____ + _____ = _____ median = _____

**Conjecture:** Whenever you add 5 consecutive integers, the sum is always _________________ the median of these numbers.

**Deductive reasoning:**

Let *any* five consecutive integers be represented by: x, x + 1, x + 2, x + 3, and x + 4

Then, the *sum* of these integers = _____________________________ = _________

The *median* of these integers = _________

5 times the median = _________

Since, in general, the sum of any five consecutive integers is equal to 5 times the median of these numbers, then our conjecture is true.
Example 2: Use Deductive Reasoning to Validate a Conjecture

Use deductive reasoning to prove the conjecture that the sum of any two odd integers will always be even.

Solution:

Deductive reasoning:

Let any two odd integers be represented by ________ and ________.

The sum of these integers = ______________________________

= _________________________

= _________________________

Since any even number is divisible by 2 and 2(x + y + 1) is divisible by 2, then 2(x + y + 1) will always be even. The conjecture that the sum of any two odd integers will always be even is true.

Example 3: Use Deductive Reasoning to Validate a Conjecture

Use deductive reasoning to prove the conjecture that the difference between two consecutive perfect squares will always be odd.

Solution:

Deductive reasoning:

Let any two consecutive perfect squares be represented by __________ and __________.

The difference of these integers = ______________________________

= _________________________

= _________________________

= _________________________

Since 2x + 1 will always be odd, then the conjecture that “the difference between two consecutive perfect squares will always be odd” is true.
Example 4: Use Deductive Reasoning to Validate a Conjecture

Jared discovered a number trick in a book he was reading:

Use inductive reasoning to make a conjecture about the relationship between the number chosen and the final result.

Use deductive reasoning to prove your conjecture.

Solution:

<table>
<thead>
<tr>
<th>Steps</th>
<th>Inductive Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose any number</td>
<td></td>
</tr>
<tr>
<td>Double it</td>
<td></td>
</tr>
<tr>
<td>Add 6</td>
<td></td>
</tr>
<tr>
<td>Double again</td>
<td></td>
</tr>
<tr>
<td>Subtract 4</td>
<td></td>
</tr>
<tr>
<td>Divide by 4</td>
<td></td>
</tr>
<tr>
<td>Subtract 2</td>
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</tbody>
</table>

Conjecture: __________________________________________________________________________

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Therefore, the conjecture is true.